EXTREME REGIMES OF TEMPERATURE AND PRESSURE IN ASTROPHYSICS

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ABSTRACT

A survey is given of states of matter and phenomena in extreme régimes of temperature and density found in the astrophysical universe: thermodynamics and stellar evolution, terminal states of stars, temperatures of stars, forms of superdense matter in stars, crystallization of superdense stellar matter, neutron star structure, and low temperature phenomena in neutron stars.

1. INTRODUCTION

We live in a remarkably special part of the universe which can sustain life: the temperature is about 300° K; the density of things around us is in the range 0·1 to 10 g cm^{-3}; we are surrounded by a gas of around 10^{-3} g cm^{-3} and pervaded by a one gauss magnetic field. Even the régimes of these parameters accessible in the laboratory cannot approach those found in parts of the astrophysical universe and even less those found in the speculations of astrophysicists.

The density of the centre of the sun, a typical hydrogen burning (main sequence) star is about 10^2 g cm^{-3}. White dwarf cores cannot exceed 10^9 g cm^{-3}. Neutron star centres are calculated to have 10^{14} to 10^{15} g cm^{-3} and may even be much denser. Greatest of all are the possible singular densities in the biography of a piece of matter. In orthodox relativity theory some world lines of our expanding universe must have originated in a singularity of infinite proper density[1–4]. For a homogeneous isotropic universe all matter had such a genesis. Finally all stars too heavy to become neutron stars will contract indefinitely; but as seen by an observer living on the inward falling surface of such a star it will rapidly pass through its Schwarzschild singularity and attain an infinite density.

The temperature in the solar centre $\sim 10^{7\circ}$K. Red giant cores are close to $10^{8\circ}$K, about the maximum achievable in the explosion of a nuclear weapon. A highly evolved heavy star, just before it becomes a supernova, reaches a central temperature near $6 \times 10^{9\circ}$K—about half a million electron volts per particle. At such a temperature black body radiation weighs 10^3 g cm^{-3} and coexists with an almost equal density of electron–positron pairs. Various scenarios which try to describe a star during a supernova explosion suggest $T \sim 10^{12\circ}$K and more.

A similar régime of extraordinarily high temperatures occurs in the orthodox general relativistic account of the creation. There is strong evidence

* Professor Ruderman was unable to be present, and a lecture based on his paper was given by Dr C. Pethick.

that the present universe contains an isotropic homogeneous sea of 2·7°K black body photon radiation. As the universe expands such radiation remains black body but with a temperature decreasing like the inverse Hubble radius of the universe. Extrapolating backward in time for a homogeneous isotropic universe gives a view like that in *Figure 1*. In the beginning there may have

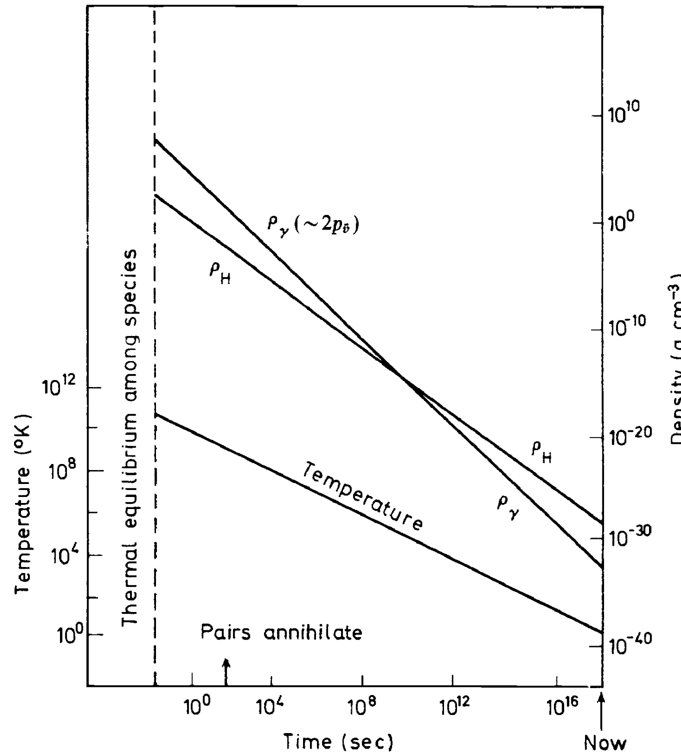


Figure 1. Temperature and density history of a homogeneous isotropic universe. ρ_H is the mass density of protons (hydrogen), $\rho_{\bar\nu}$ that of photons, and $\rho_{\nu-}$ that of (anti) neutrinos. Details in ref. 5.

been thermal equilibrium for a fraction of a second. Baryons, antibaryons, mesons, neutrinos, electrons, perhaps quarks or magnetic poles, would have coexisted but after a second or so, when the initial large temperature has fallen to near 10^{10}°K, the weakly interacting neutrinos no longer interact significantly with the remaining matter and are 'frozen out' of the equilibrium. A measurable fraction of quarks and antiquarks ($\sim 10^{-20}$ quarks/nucleon) may remain[5]. Even nuclear reactions cease 10^2 seconds after the big bang when the temperature has dropped to 10^9°K. Electron–positron annihilation then eliminates all positrons and most electrons. The expanding universe then consists of protons, electrons and neutrinos; a considerable fraction of helium ($\sim$20 per cent by mass), and very small fractions of the lighter elements (D, ^{3}He, ^{3}H, ^{7}Be, ^{7}Li).

The very earliest stage, where local thermal equilibrium is presumed to prevail, encourages speculations about possible phenomena that might affect the subsequent evolution of the universe. Some concern mechanisms for establishing the truly remarkable isotropy[6] we see in present cosmic black body radiation[7] from essentially random initial conditions[8]. An extremely vital and entertaining thermodynamic question is the possibility of phase separations. Could a dense universe of nucleons, mesons and antinucleons or perhaps even quarks and antiquarks have spontaneous macroscopically separated phases which partially separate matter from antimatter[9]? The signs and magnitudes of the interactions among nucleons and mesons are indeed such that at certain densities ($\rho \sim 10^{12}$ g cm^{-3}) and temperatures ($T \sim 10^{12}$°K) the free energy might conceivably be lowered by a two phase system one of which is primarily baryons and those mesons attracted to them, the other the charge conjugate, i.e. anti-system. However, there is no adequate quantitative argument that this should be so[10].

Unfortunately the very early stages of the universe are a régime in which not only may the relevant physical laws possibly be incomplete, but the initial conditions almost certainly are. However, the extreme early stages of matter, enormous temperatures, densities, magnetic fields etc. probably are duplicated in the evolution of many stars—only the time sequence being opposite to that of an expanding universe: the star evolves from a hot diffuse gas through ever denser and hotter régimes to one of a number of enormously dense final states. Remarkably the final stages in the death of a star involve at first the highest temperatures known to exist in our present universe; then, usually there follows an extraordinarily rapid cooling which effectively, i.e. in terms of the phenomena which occur, is to a far lower temperature than can be found anywhere else in the natural universe.

2. THERMODYNAMICS AND STELLAR EVOLUTION

In the beginning a star is a mass of gas held in quasi-static equilibrium with the pull of gravity balanced by classical kinetic pressure. Even if it did not radiate it would never be in true thermodynamic equilibrium. A system of classical particles interacting through coulomb and gravitational forces has no lowest energy state. It continually evaporates particles, sending some to infinity and lowering the energy of the remainder. The time scale for such motions is usually sufficiently slow that they are neglected in analyses of stellar structure, at least in earlier stages of stellar evolution. The thermodynamicist might describe such a quasi-static star as a system of negative specific heat. Thirring[11] has recently emphasized that *any* condensed (i.e. bound) system of classical particles which interact among themselves through electrical and gravitational forces should behave as if its specific heat were negative. The total energy

$$E = K + V \tag{1}$$

where K is the total kinetic and V the potential energy of the particle system. For inverse square law forces the virial theorem gives

$$K = -\tfrac{1}{2}V \tag{2}$$

so that

$$E = -K \equiv -\tfrac{3}{2}Nk\langle T\rangle \tag{3}$$

Here N is the particle number and $\langle T \rangle$ the appropriate average temperature. Then the heat capacity $\langle c \rangle$ is defined by

$$\langle c \rangle \equiv \partial E/\partial \langle T \rangle = -\tfrac{3}{2}Nk < 0 \tag{4}$$

A more detailed analysis[11] which includes the virial effects of boundaries shows that both the core of a star and the outer parts separately behave as if they had negative heat capacity. Two systems in thermal contact have no equilibrium when each of them has negative heat capacity. The change in entropy $S(E)$ when a small amount of energy ε is transferred from system 1 at temperature T_1 to system 2 at temperature T_2 is

$$\delta S = \varepsilon \left\{ \frac{1}{T_1} - \frac{1}{T_2} \right\} - \frac{\varepsilon^2}{2} \left\{ \frac{1}{c_1 T_1^2} + \frac{1}{c_2 T_2^2} \right\} \tag{5}$$

where

$$S = S_1(E_1) + S_2(E - E_1) \tag{6}$$

and

$$\frac{1}{T_i} = \frac{\partial S_i}{\partial E_i} \quad \frac{1}{c_i} = -T_i^2 \frac{\partial^2 S_i}{\partial E_i^2} \tag{7}$$

When $T_1 = T_2$, stability, i.e. $\delta S < 0$, follows only when $(c_1)^{-1} + (c_2)^{-1} > 0$. When both heat capacities are negative the hotter system 1 (the stellar core) transfers energy to the cooler system 2 (the stellar exterior). The core grows continually hotter, the exterior continually cooler [cf. *Figure 2*]. The virial

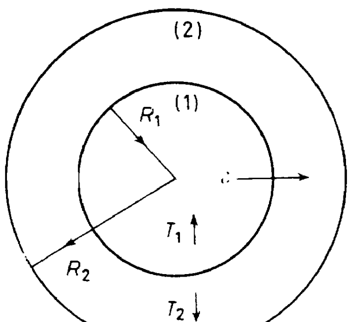


Figure 2. Schematic representation of two negative heat capacity regions of a star.

theorem relates the core temperature T_c to its average density ρ_c. The evolution of a stellar core, as long as it can be described as a system of classical particles, is then approximately given by

$$\rho_c \sim T_c^3 \tag{8}$$

Figure 3 gives the computed evolutionary track for a 16 solar mass ($M_\odot$) star's core as computed by Hayashi, Sogi and Sugimoto[12]. It is well approximated by equation 8. While the core contracts and heats, the outer stellar region cools (since its heat capacity is also negative) and, according to the virial theorem, expands. Thus the evolutionary trend of stellar envelopes is toward cooler, larger surfaces. This is confirmed in the familiar Hertzsprung–Russell diagram for the stellar observables: surface temperature T_e and luminosity L.

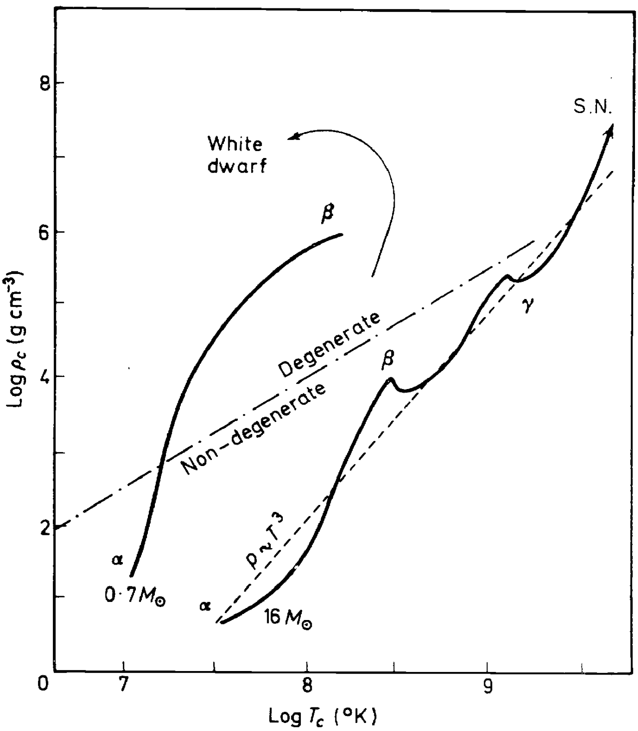


Figure 3. Evolution of stellar cores. α—hydrogen burning; β—helium burning (ref. 12); γ—C, O, Ne burning points. The lighter star ($M = 0{\cdot}7\ M_\odot$) quickly becomes degenerate after it leaves the main sequence and evolves very differently.

In *Figure 4* we see that the evolutionary trend of a 16 $M_\odot$ star is toward lower surface temperature. Since the stellar radius R_e depends upon L and T_e approximately according to the black body prescription

$$L \sim R_e^2 T_e^4 \tag{9}$$

the star expands greatly as it moves to the right off the main sequence, beginning as a blue star and ending as a red supergiant. Its outer radius expands by a factor of 100 as the atmosphere cools; its core radius decreases by a factor of 30 as the core temperature rises toward 10^9°K. A similar evolution—that of two systems of negative heat capacity in thermal contact—may ultimately obtain for entire galaxies[13]. The galactic core of stars shrinks as the stars within it acquire more kinetic energy while the rest of the galaxy expands as the container stars slow up. However, the stellar mean free path is so long that such a qualitative separation into two stellar systems is not maintained. A possible relationship between the expected thermodynamic behaviour of the central parts of classical stellar systems bound by inverse square law forces and observations of very dense very active galactic cores is still obscure.

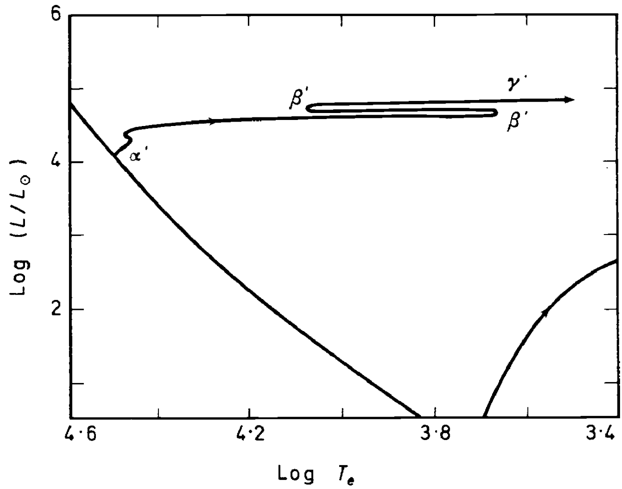


Figure 4. Evolution of stellar surfaces (ref. 12). The points α′, β′, γ′ correspond to the interior points α, β, γ of *Figure 3.* The diagonal line is the main sequence for H-burning stars of all masses. The lower right hand segment corresponds to a 0·7 $M_{\odot}$ star.

3. TERMINAL STATES OF STARS

The evolution of a stellar core through stages of ever increasing densities and temperatures is stopped temporarily by the ignition and consumption of nuclear fuels. During such stages, which occupy most of the stellar lifetime, there is a quite stable balance between energy radiated, energy transferred from the core to the envelope, and energy generated within the core (or its boundary) by exothermal nuclear processes. But the ultimate asymptotic state of a star is achieved only by mechanisms which can permanently stop its contraction. There are three known terminal stellar states: white dwarfs, neutron stars, and gravitational collapse to black holes.

As a stellar core contracts its temperature rises like $\rho^{1/3}$. But the degeneracy energy of non-relativistic electrons rises like $\rho^{2/3}$ (the kinetic motion of electrons contributes most of the central pressure) so that these electrons can ultimately become degenerate. When this happens the kinetic energy of the electrons and therefore the core pressure no longer depends sensitively upon temperature. Such degenerate stars behave like normal condensed laboratory matter where quantum mechanical electron degeneracy is basically responsible for stability. The virial theorem no longer implies a negative heat capacity, and the star core cools as energy is radiated.

Quite generally stable stars which can be the end states of stellar evolution are composed of such cool matter in which thermal pressure makes a negligible contribution. The source of pressure may be electron degeneracy (white dwarfs) or nucleon repulsion when the central density is sufficiently large (neutron stars).

The equation of state of cool matter over all régimes relevant for stellar matter is sketched in *Figure 5*. Only a very small part of it, corresponding to

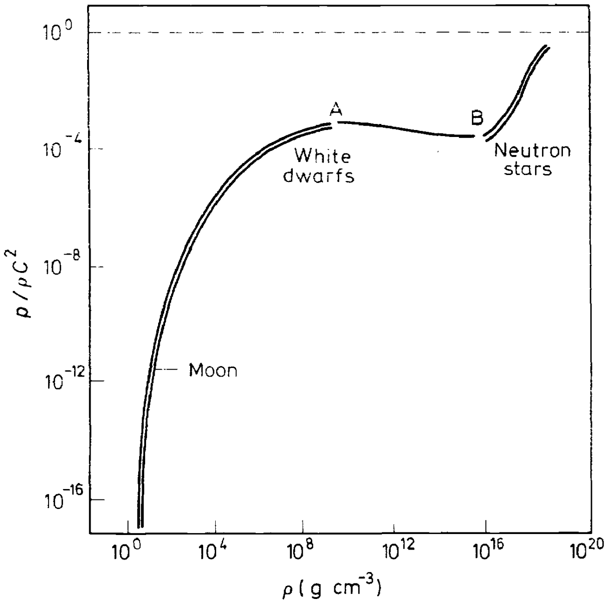


Figure 5. The ratio of pressure (p) to density (ρ) times c^2 as a function of density for cool matter.

matter at the density of the moon's centre is accessible in the laboratory. Above 10^5 g cm^{-3} the electrons which are the main contributor to the pressure are well approximated by a non-relativistic degenerate electron gas. At $\rho \sim 10^7$ g cm^{-3} the electron Fermi energy approaches 10^6 eV and the electrons become relativistic. The matter is then less stiff since the pressure of relativistic degenerate electrons does not rise as rapidly with decreasing volume as does that of non-relativistic ones. As the electron Fermi energy rises above 1 MeV ($\rho \gtrsim 10^7$ g cm^{-3}) the embedded nuclei become unstable against the capture of energetic electrons which convert nuclear protons to neutrons plus neutrinos which readily escape from the star without further interaction. In such a régime the matter is quite compressible since the electron Fermi energy (and hence pressure) ceases to rise substantially with increasing density. Rather the electrons continue to be absorbed by protons until at $\rho \sim 5 \times 10^{13}$ g cm^{-3} all but a few per cent of the nucleons are neutrons. Subsequent increases in density give a rapidly rising pressure in part because of non-relativistic neutron degeneracy and more because of strong short range neutron repulsion. For $\rho \gtrsim 10^{15}$ g cm^{-3} the interaction energy among particles is no longer small next to the mass differences among elementary particles or even next to the total rest masses. Here nothing is really known about the appropriate equation of state for matter. The restriction

$$p < \rho c^2 \tag{10}$$

is necessary if the sound speed at long wavelengths

$$c_s^2 = \mathrm{d}p/\mathrm{d}\rho \tag{11}$$

is to be less than the speed of light. Zel'dovich[14] has suggested a model that attains the high density limit $p = \rho c^2$. However, this limit may not have a sacred role in theoretical physics. Violations do not contradict Lorentz invariance and positive definiteness of energy[15]. But various proposed theories which do permit $p > \rho c^2$ have all involved either an unacceptable instability or a violation of conventional notions of causality[16,17]. There are of course no experimental data to test either the high density limit of the equation of state or the validity of causality in such a régime.

That part of the equation of state curve of *Figure 5* which is doubled corresponds to stiff matter which satisfies the criterion

$$\mathrm{d}p/\mathrm{d}\rho > \tfrac{4}{3}p/\rho \tag{12}$$

It consists of two disjoint sections. Such matter is stiff enough to constitute the core of cool stable stars which are able to resist being crushed by their own gravitational contractions. The associated stars are represented in *Figure 6*. Dying stars associated with the lower segment of the equation of state curve are white dwarfs and planets. Stars corresponding to the upper segment are called neutron stars even if the main constituents of the core (for $\rho \gtrsim 10^{15}$ g cm^{-3}) are no longer neutrons.

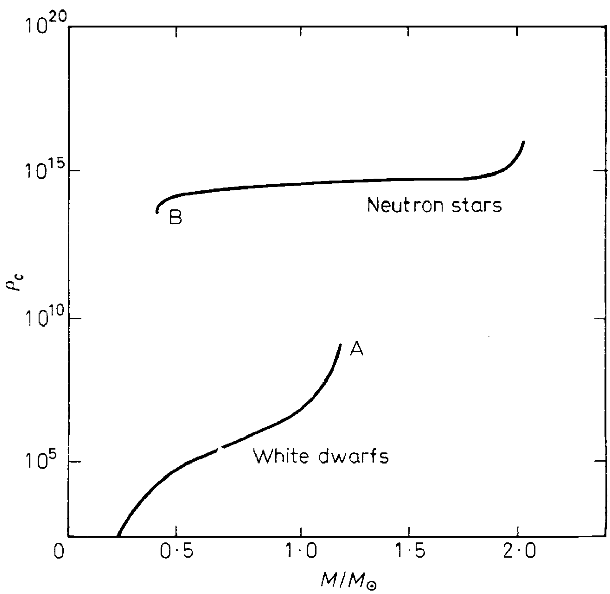


Figure 6. Masses of cool stars, in units of the solar mass $M_\odot$, as a function of central density ρ_c. The points A and B correspond to those of *Figure 5*.

There is an upper bound of about two solar masses ($M_\odot$) above which no cool stable star can exist. Even if matter were to become infinitely stiff ($dp/d\rho \to \infty$), this limit would not substantially change since increased pressure also increases the gravitational attraction between various parts of the star sufficiently to crush the star no matter how stiff it is. Thus if $M \gtrsim 2M_\odot$ the star cannot die as a stable star. Nothing can permanently stop its continual contraction toward infinite density. After the termination of its nuclear burning a heavy star of modest angular momentum will collapse through its Schwarzschild singularity radius ($R \sim 2\,GM\,c^{-2} \sim 10^6$ cm) until at least parts of it reach infinite density[1]. (Quantum corrections to orthodox general relativity might conceivably change this conclusion but not before $\rho \sim 10^{90}$ g cm^{-3}). As seen by an observer on the contracting star the collapse through the Schwarzschild radius and to a singularity takes only a few seconds. But a distant observer sees a star asymptotically approaching the Schwarzschild radius which it never reaches. The light from the stellar surface grows redder and dimmer. The singular densities which it can attain inside the Schwarzschild radius can, in orthodox general relativity, have no observable consequences outside. We shall concern ourselves only with the thermodynamic nature of the states associated with the super-high densities and unique temperatures within the white dwarfs and neutron stars.

4. HOW HOT DO STARS GET?

A star which does not end its life as a white dwarf will probably reach a stage where all the core nuclear constituents have ignited leaving behind only iron peak elements which have the highest binding energy of all nuclear species. This will occur at core temperatures $T_c \sim 5 \times 10^{9}$°K and correspondingly high core densities of perhaps 10^8 g cm^{-3}. The chief mechanism for energy transfer out of such a core is neutrino pair emission from electron–positron recombination. The neutrino emission of one such star $\sim 10^{46}$ ergs/sec greatly exceeds the light emission from the entire galaxy. Such stellar cores become unstable a few minutes after the high central temperature is reached: the core contraction rate, limited by the free fall rate of core matter, is too slow to supply the enormous amounts of energy which must be supplied to the core to maintain quasi-equilibrium. Two effects contribute to this negative energy budget. As the temperature rises the minimum free energy $U - TS$ is achieved not by minimizing U which resulted in burning all nuclei to iron but rather in maximizing the entropy S which is attained by breaking up the iron into its constituent α particles and neutrons. This endothermic undoing of the previous nuclear fusion occurs very rapidly and acts as a refrigerant in the core[18]. Simultaneously the neutrino pair emission rate removes energy more rapidly than the core contraction supplies it[19]. The core then is imploding in almost free fall. The neutrino pair emission processes keep it sufficiently cool[20] that the decreasing gravitational potential energy is converted into inward radial velocity rather than thermal energy. If nothing were to stop the collapse such a star would probably not heat greatly as it approached its Schwarzschild radius. But at least for the lighter stars ($M \lesssim 2M_\odot$) nuclear repulsion can stop the collapse and the resulting neutron star is formed with internal kinetic energies of about 0·1 Mc^2. For the classical heat capacity of free neutrons this implies a temperature $T \sim 10^{12}$°K. Aside from the very problematical initial moments of the universe this is the hottest conjectured temperature in any object in the universe.

It has been argued that 10^{12}°K is the highest temperature that can ever be achieved anywhere[21, 22]. This thesis is based upon some considerable successes of statistical models in elementary particle physics: the centre of mass energy in a very high energy particle collision is confined for about 10^{-23} sec in a very small interaction volume. This is conjectured to be long enough for thermal equilibrium to obtain. Various features of such collisions, for example the exp $(-p_\perp c/kT)$ distribution of transverse momenta $p_\perp$ for emitted particles, support such a picture. Non-relativistically $T \sim E_{CM}$; for dominant particle creation, as in black body radiation, $T \sim E_{CM}^{\frac{1}{4}}$; experimentally $T \sim T_0 - KE_{CM}^{-1}$ with $T_0 \sim 1{\cdot}2 \times 10^{12}$°K. Hagedorn[22] has proposed that such a limit, $T < T_0$, follows very naturally from the enormous (exponential) proliferation of new particles and resonances with increasing mass. The number of degrees of freedom in his model increases so rapidly that for thermal equilibrium the energy density ε approaches $\varepsilon = AT_0(T_0 - T)^{-1}$ so that $T_0 = 1{\cdot}2 \times 10^{12}$°K is never exceeded.

No matter what the initial high temperature of a neutron star it will cool by neutrino emission extremely rapidly—to well below 10^{10}°K in less than a second and to near 10^8°K within 10^3 years. The very peculiar effects of the huge neutron star magnetic fields upon stellar atmosphere opacity and radiation processes will probably greatly accelerate the stellar cooling, but no quantitative calculations of the cooling rates have been presented.

There is abundant circumstantial evidence that pulsars are rotating neutron stars. The youngest pulsar, the central star of the Crab Nebula, is a remnant of a thousand year old supernova explosion; this neutron star—once the hottest object in the universe—has now cooled so that its interior temperature is of order 10^8°K. This is about ten times hotter than the solar centre but we shall see that in terms of phenomena it is now probably one of the coldest places in the universe.

5. FORMS OF SUPERDENSE MATTER IN STARS[23]

The régimes for various forms of superdense matter are sketched in *Figure* 7. At $T \sim 10^8$°K and below it is generally crystalline or (above $\rho \sim 10^{14}$ g cm^{-3}) mainly neutron superfluid. A walk into a white dwarf from its surface to its centre begins in a gaseous atmosphere and ends, often, in a solid with $T_c \sim 10^7$°K and $\rho_c \sim 10^5$ to 10^8 g cm^{-3}. A similar stroll into a neutron star passes through a much more varied environment [*Figure 8*]. The atmosphere is a few centimetres thick. A few metres below the surface the electrons are highly degenerate and very good thermal conductors so that the star has a constant temperature from that point up to the centre 10^6 cm away. The nuclei arrange themselves into a crystal so that the star has a solid crust usually a kilometre or so thick. The crust disappears because the nuclei do at $\rho \sim 5 \times 10^{13}$ g cm^{-3}. Below this the predominant neutrons

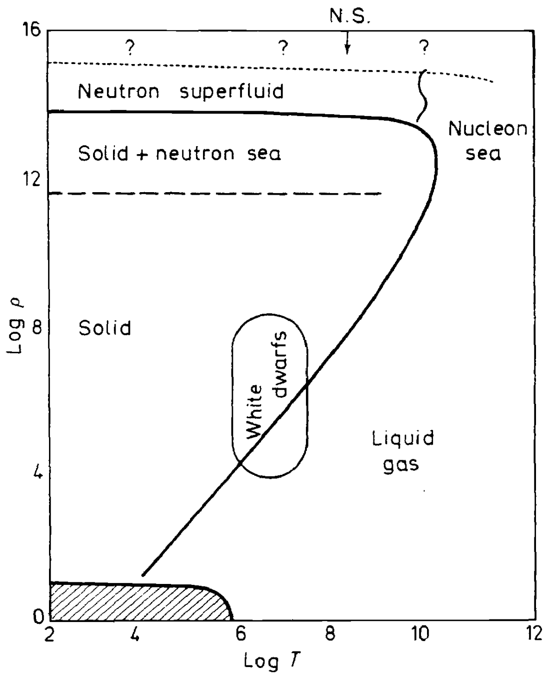


Figure 7. Forms of matter for extreme regions of temperature and pressure. White dwarf core regions are explicitly designated. The crosshatched region is about that attainable in the laboratory.

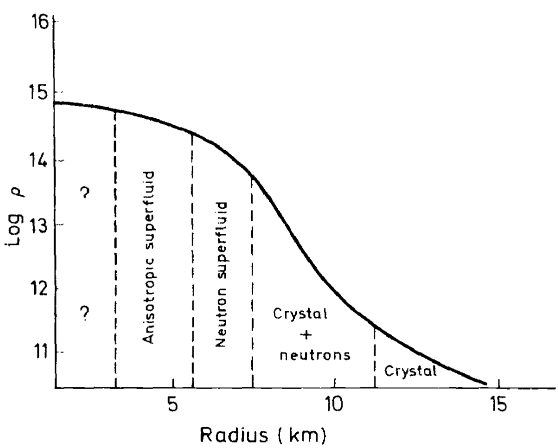


Figure 8. Forms of matter and density in parts of a light neutron star.

are probably a superfluid analogous to the Bardeen–Cooper–Schrieffer electron paired superconductor except that the paired neutrons carry no charge. At about $\rho \sim 3 \times 10^{14}$ g cm^{-3} this superfluid is predicted to take an anisotropic form which has not yet been seen in any laboratory superfluid. As ρ approaches 10^{15} g cm^{-3}, which will happen at the centre of many and perhaps even most neutron stars, the form of the matter, its constituents and the equation of state are not yet known.

6. CRYSTALLIZATION OF SUPERDENSE STELLAR MATTER[24–28]

The core of white dwarfs and the outer regions of neutron stars consist of qualitatively similar material—a highly degenerate electron sea in which are embedded nuclei. A white dwarf's history has probably been such that the matter within it has not yet burned to iron but all helium has been converted to heavier elements; therefore, for the embedded nuclei $2 < Z < 26$. The presumed genesis of neutron stars suggests that all the matter has its lowest free energy; the resulting $Z \geqslant 26$ depends upon the local electron Fermi energy.

When nuclei are embedded in a degenerate electron sea their coulomb fields are screened by the surrounding electrons. In normal laboratory matter almost all of the nuclear charge is completely screened so that the correlation energy between nearest neighbour nuclei is only a few electron volts, characteristic of that for single net charges. The relevant interaction energy

$$\frac{(Ze)^2}{r_z} \rightarrow \frac{(Ze)^2}{r_z} \exp\left(-\frac{r_z}{r_{sc}}\right) + \text{Osc.} \tag{13}$$

where r_z is the separation between neighbouring nuclei, r_{sc} the screening radius (Debye radius), and Osc. a small oscillating term. In laboratory matter $r_{sc} \ll r_z$, but as matter is squeezed to much higher densities the Fermi energy increases so much that the electron kinetic energy exceeds not only the electron–electron energy but also the larger interaction between the electrons and the coulomb fields of the nuclei. (Non-relativistic Fermi energies grow like $\rho^{\frac{2}{3}}$ while interaction energies increase less rapidly, like $\rho^{\frac{1}{3}}$.) Therefore with increasing density the nuclear coulomb fields exert an ever smaller perturbation upon the high momentum electrons around them. When the electron Fermi energy exceeds 10^6 eV ($\rho \lesssim 10^7$ g cm^{-3}), the screening radius $r_{sc} \approx 6\, r_z\, Z^{-\frac{1}{3}}$. Even for iron ($Z = 26$) nearest neighbours and even next nearest neighbours see essentially the unscreened coulomb fields of surrounding nuclei. Over short distances the electron sea behaves like an unpolarizable uniform negative background in which the bare nuclei are embedded. The correlation energies among nuclei become quite enormous. Thus for iron with $\rho \sim 10^8$ g cm^{-3} (the centre of a rather heavy white dwarf) the repulsive coulomb energy is about 10^6 eV, about a million times greater than that for iron at normal densities. It is this enormous correlation energy which causes crystallization of superdense matter even at a temperature of many hundreds of millions of degrees.

If the electron screening is neglected completely, the pure coulomb interaction among nuclei gives a system which has been worked on extensively for thirty five years. At zero temperature the nuclei form a body-centred cubic lattice whose thermal properties have been extensively tabulated. Because the theories of melting are so far from being definitive, mostly because the solid/liquid transition does not greatly change the properties of matter, the melting temperature of such a lattice is not well known. Dimensionally

$$kT_m = (1/\Gamma)(Ze)^2/r_z \tag{14}$$

where Γ is a pure number, independent of the charge Ze and of the nuclear separation r_z. Thus if Γ is known for any coulomb lattice it is known for all. Most substances melt when their interaction energy is about one per cent of the particle interaction energy so that $\Gamma \sim 10^2$. A rough estimate[28] applying Lindemann's rule to the lattice excitations gives $\Gamma \sim 60$. Van Horn[26] estimates $\Gamma \sim 52$ for a theoretical model and 150 from a more precise application of Lindemann's rule[29]. A computer experiment[30] on a classical coulomb gas of 32 particles indicated an instability at $\Gamma = 126$ which resembled the finite particle analogue of a phase transition. Then for white dwarf matter

$$T_m \sim 10^7 (\rho/10^6)^{\frac{1}{3}} (Z/8)^{\frac{5}{3}} \,^\circ\mathrm{K} \tag{15}$$

Typical white dwarf centres have $T_c \sim 10^{7}\,^\circ$K, close to their melting temperatures. An unanswered question which may have some observational consequences for white dwarfs is whether in this high density régime the melting transition remains a first order one and the size of the heat of transition if it is. A significant heat of fusion ($\sim kT_m$ per nucleus) might observably retard white dwarf cooling rates in the transition region.

7. NEUTRON STAR STRUCTURE

The composition of matter in some density régimes relevant to neutron star interiors is given[31] in *Figure 9*. (It is assumed that the matter is in its lowest energy state.) The numerical density of nuclei, n_z, varies only between 10^{33} and 10^{34} cm^{-3} as ρ varies from 10^{11} to 5×10^{13} g cm^{-3}. The nuclei arrange themselves in a lattice whose melting temperature is given[32] in *Figure 10*. Also plotted is the crystal Debye temperature. Since the expected temperature inside the neutron star is at most a few times $10^{8}\,^\circ$K, the outer neutron star layer is very far below its melting temperature; it is also a quantum solid with very little heat content. The neutron star 'crust' is much more solid than that of the earth: its terrestrial analogue would be a thick shell of iron at a temperature of at most a few tens of degrees. Inhomogeneities from various frozen-in nuclear species and possible phase separations may give a rich 'geology' to such cold crusts.

At $\rho \sim 5 \times 10^{11}$ g cm^{-3} free neutrons coexist with the nuclei. With increasing density the fraction of neutrons which are free increases until $\rho \sim 5 \times 10^{13}$ g cm^{-3}. There all nuclei disappear and the neutrons constitute a degenerate neutron sea whose Fermi energy $\sim$20 MeV. Pairs of neutrons at the top of the Fermi sea attract each other. Relevant neutron–neutron

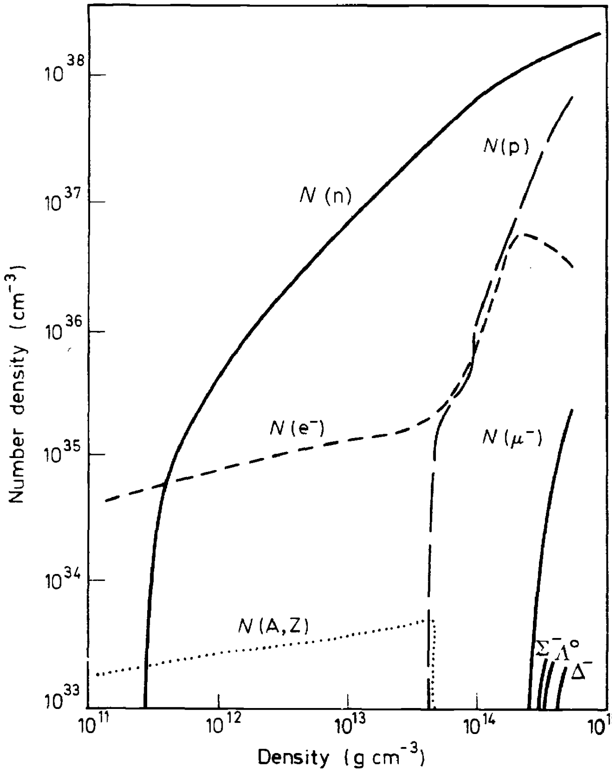


Figure 9. Constituents of 'cold' matter as a function of density (ref. 31).

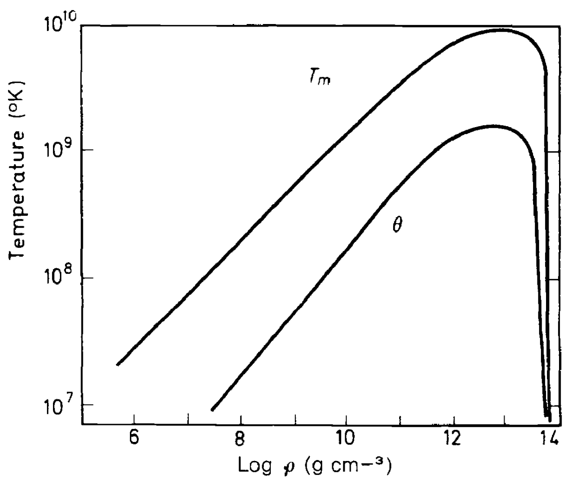
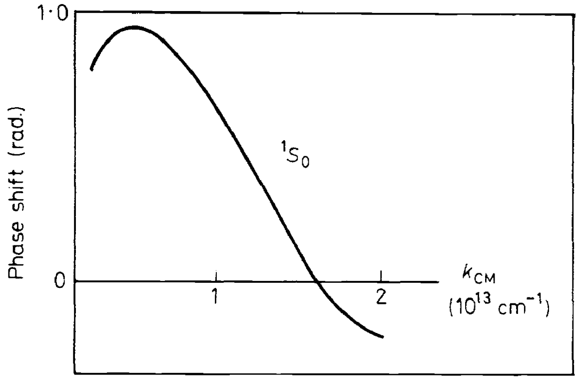


Figure 10. Melting temperature T_m and Debye temperature θ for matter at various densities. Both temperatures drop discontinuously where $N(A, Z)$ does.

scattering phase shifts are represented in *Figure 11* as a function of the wave number of either neutron in the centre of mass system. The $^1S_0$ phase shift is attractive for $k_{CM} < 1{\cdot}4 \times 10^{13}$ cm^{-1} corresponding to $\rho \lesssim 1{\cdot}5 \times 10^{14}$ g cm^{-3}. At higher densities this phase shift becomes repulsive and only the $^3P_2$ interaction remains attractive. According to the Bardeen–Cooper–



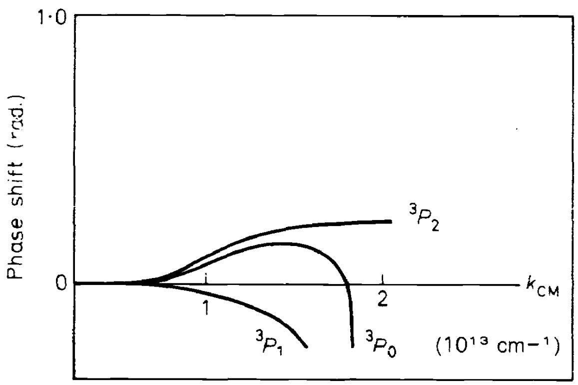


Figure 11. Phase shifts for neutron–neutron scattering as obtained from laboratory proton–proton scattering experiments. k_{CM} is the wave number of either neutron in the centre of mass frame.

Schrieffer theory of electron superconductivity any attractive phase shift at the top of the Fermi sea is sufficient to give a gap in the single particle excitation spectrum. The gap gives superfluid (but not superconducting) properties to the neutrons[33–37]. An estimate of the transition temperature into this state[35–37], based upon the phase shifts of *Figure 11*, is given in *Figure 12*.The computed superfluid transition temperatures are much more than an order of magnitude greater than the expected temperature within the neutron star. Therefore the stellar interior should be filled with a very 'cold' neutron superfluid. Below $\rho \sim 1{\cdot}5 \times 10^{14}$ g cm^{-3} such a superfluid has conventional properties like that of superfluid ^{4}He. At higher densitie where the neutron pairing is attractive only in a $J = 2$ state the superflui gap is anisotropic; $\Delta \sim (\frac{1}{3} + \cos^2 \theta)$. There will be an anisotropic compressi- bility associated with this gap. The direction $\theta = 0$ is determined by internal stresses within the star to be in the radial direction.

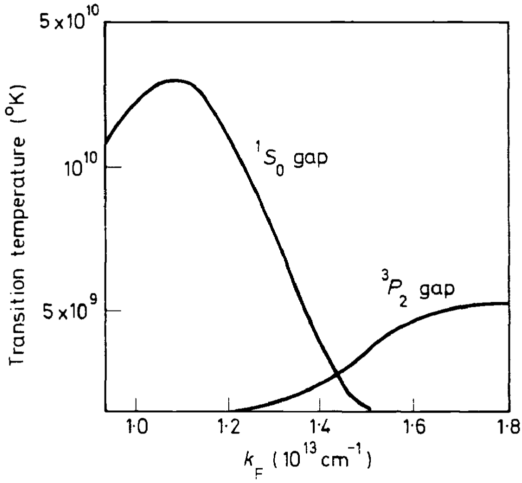


Figure 12. Estimated transition temperatures into the superfluid state for a degenerate neutron sea as a function of k_f, the wave number at the top of the sea (refs. 35–37).

Coexisting with the superfluid neutrons are degenerate seas of electrons and protons, numerically a few per cent as dense as that of the neutrons. The Fermi energy of the degenerate electrons must be just sufficient to prevent both the decay $n \rightarrow p + e + \bar{\nu}$ and its inverse $e + p \rightarrow n + \nu$, otherwise the neutron sea would be unstable. The needed electron Fermi energy, E_F, is about 10^2 MeV; because the electrons are light and relativistic, their numerical density, and that of the protons, is much less than that of the neutrons by a factor $(E_F/2m_nc^2)^{\frac{3}{2}}$. These highly relativistic electrons are an extremely good electrical conductor (neutron star internal magnetic fields have a decay time $\sim 10^{13}$ years)[38]. The protons are likely to be superconducting. Although the few protons can be ignored in describing the interaction between neutron pairs at the top of their Fermi sea the converse is not true. If, however, the neutron sea polarizability is neglected and the free proton–proton interaction is used, predicted proton superconducting transition temperatures are in excess of 10^9°K. The superconducting protons do not expel the large ($\sim 10^{12}$ gauss?) neutron star magnetic fields but rather form a type II superconductor which channels the magnetic field within it[39].

In the lighter neutron stars the superfluid-superconducting matter extends to the centre. In the heavier ones the central density reaches and exceeds 10^{15} g cm^{-3}. If interactions are ignored the lowest energy state compatible with the Pauli principle consists of electrons, protons and neutrons together with μ-mesons, hyperons, resonances etc. The interaction energie between particles are comparable to the mass differences and even the enti rest masses. The properties of such a relativistic conglomerate are unknown and may be quite peculiar.

8. LOW TEMPERATURE PHENOMENA IN NEUTRON STARS

The terrestrial laboratory analogue of a neutron star is a very cold thick spherical iron shell filled with superfluid helium and a possible unknown central core. The liquid helium would be at less than a few $\times 10^{-2}$°K, fantastically cold, in terms of its transition temperature. The neutron fluid is almost incompressible; its sound speed is $10^{-1}c$. Therefore at $T \sim 10^{8}$°K there are almost no phonons, the fraction of the fluid which in the canonical nomenclature is 'normal' (ρ_n/ρ), being only 10^{-14}. The analogous ratio for laboratory superfluid helium is attained at well below 10^{-2}°K. The neutron star interior, if it is indeed such a 'cold' superfluid, has a very small heat capacity which resides mainly in the degenerate electrons, about 10^{-6} the heat capacity of a classical neutron gas of equal density. At 10^{8}°K the neutron star interior is phenomenologically the coldest known place in the universe.

The association of rotating neutron stars with pulsars offers the possibility of actually observing in neutron stars phenomena unique to very low temperature rotating superfluids. The precision with which pulsar periods can be measured, especially in the short period Crab and Vela pulsars, permits the detection of angular velocity variations of less than 1 in 10^{10} (corresponding to, say, a change in moment of inertia caused by a variation in shape or radius of 10^{-4} cm). Discontinuous increases have been observed in the angular frequency (Ω) of the Vela pulsar ($\Delta\Omega/\Omega = 2 \times 10^{-6}$)[40–41] and the Crab pulsar ($\Delta\Omega/\Omega = 4 \times 10^{-9}$)[42] which are very likely the result of some sudden event which slightly speeds up the spinning crust (starquake?). As the increased crust angular velocity is shared with the much more massive superfluid neutron interior Ω tends to return toward what it would have been without the discontinuity. The time scale for this 'healing' of the crust–interior angular frequency mismatch is of order a few years in the Vela pulsar. These long spin up times are characteristic of those calculated for cold superfluid interiors[43]. The highly conducting crust is coupled to the interior electron–proton sea by the very strong magnetic field which is presumed to pervade the entire star; therefore the electrons and protons co-rotate with the crust. If the protons and neutrons were not superfluid the interaction time between them would be only of order 10^{-15} sec because of proton–neutron collisions. But such transfers of momentum to individual neutrons is essentially forbidden in the superfluid state. However, the rotating neutron superfluid must flow irrotationally: it can mimic rigid body rotation through a paraxial array of moving quantized vortex lines. The fluid velocity satisfies $\nabla \times \mathbf{v} = 0$ except at the centre of each vortex. For rotating neutron star interiors the core radii $\sim 10^{-12}$ cm and the separation between quantized vortex lines is $\sim 10^{-2}$ cm. Only when averaged over many vortex lines can the average fluid velocity $\langle \mathbf{v} \rangle$ satisfy the usual rigid rotation relation $\nabla \times \langle \mathbf{v} \rangle = 2\boldsymbol{\Omega}$. Those neutrons within the vortex cores can still absorb momenta from p–n or e–n collisions, but they are only 10^{-18} of all the neutrons present. When both protons and neutrons are cold superfluids the main mechanism for coupling between the rotating electron–proton sea and the rotating neutrons is collisions of the relativistic electrons with the neutron magnetic moments. The characteristic time for interaction depends sensitively upon the temperature and the gap energy but is characteristically of order a year for $T \sim 10^8$ °K. If the protons are not superfluid the interaction time is reduced by about 10^6. The above model for the apparent 'healing time' after the pulsar frequency jumps is tenable only with superfluid neutron interiors.

A further possibility for observing effects of the cold rotating neutron superfluid is through the excitation of normal modes of the vortex lattice array. In the co-rotating reference frame the superfluid free energy is minimized (for cylindrical geometry) by a regular triangular lattice array at rest with respect to the container walls. There exists one very slow excitation mode of the array in which each vortex line remains parallel to the (cylinder) axis but there is a redistribution of density of vortices and superfluid angular momentum. Tkachenko[44] has shown that the displacements, **s**, of the vortex lines from their equilibrium positions satisfy a wave equation

$$\ddot{\mathbf{s}} - C_v^2 \nabla^2 \mathbf{s} = 0 \tag{16}$$

with wave velocity

$$c_v = (\hbar \Omega / 8 m_n)^{\frac{1}{2}} \sim 1 \text{ cm sec}^{-1} \tag{17}$$

(m_n is the neutron mass). Dyson[45] has shown that a necessary subsidiary condition is

$$\nabla \times \dot{\mathbf{s}} = -2\Omega \nabla \cdot \mathbf{s} \tag{18}$$

The Tkachenko–Dyson equations give a fundamental mode[46] whose period τ is proportional to the neutron star radius ($R \sim 10^6$ cm)

$$\tau \sim 140 R / \Omega^{\frac{1}{2}} \tag{19}$$

This period is close to one of a few months reported[47] for a very small wobble in the Crab pulsar ($\Omega \sim 200$ sec^{-1}). No other normal mode seems to have such a long period, but free nutations from small deformations supported by crust rigidity, or planet induced motions, or even an artifact of the necessarily marginal data analysis may account for the observations. It would be amusing if a new low temperature 'sound' phenomenon not yet seen in the laboratory were first discovered in the coldest natural place in the universe, the 10^8 °K interior of a neutron star.

REFERENCES

1 R. Penrose, *Phys. Rev. Letters*, **14**, 57 (1965).
2 S. Hawking, *Phys. Rev. Letters*, **15**, 689 (1965).
3 S. Hawking and G. Ellis, *Phys. Rev. Letters*, **17**, 246 (1965).
S. Hawking and G. Ellis, *Astrophys. J.* **152**, 25 (1968).
4 S. Hawking and D. Sciama, *Comments on Astrophysics*, **1**, 1 (1969).
5 R. Wagoner, W. Fowler and F. Hoyle, *Astrophys. J.* **148**, 3 (1967).
6 Ya. Zel'dovich, L. Okun' and S. Pikel'ner, *Soviet Phys. Uspekhi*, **8**, 702 (1966).
7 R. Partridge and D. Wilkinson, *Phys. Rev. Letters*, **18**, 557 (1967).
8 C. Misner, *Nature, London*, **214**, 40 (1967).
9 R. Omnès, *Journal de Physique*, **30**, Suppl. C3 (1969).

[10] C. Bouchiat, ORSAY preprint.
[11] W. Thirring, CERN preprint.
[12] C. Hayashi, R. Hoshi and D. Sugimoto, *Progr. Theor. Phys.—Suppl.* **22** (1962).
[13] D. Lynden-Bell and R. Wood, *Mon. Not. R. Astron. Soc.* **138**, 495 (1968).
[14] Ya. Zel'dovich, *Soviet Phys. JETP*, **14**, 1143 (1962).
[15] S. Bludman and M. Ruderman, *Phys. Rev.* **170**, 1176 (1968).
[16] M. Ruderman, *Phys. Rev.* **172**, 1286 (1968).
[17] S. Coleman, Preprint of Lectures of 1969 International School of Physics, "Ettore Majorana" (1969).
[18] F. Hoyle and W. Fowler, *Ap. J.* **132**, 565 (1960).
[19] H-Y. Chiu, *Ann. Phys.* (*N.Y.*), **26**, 364 (1964).
[20] F. Hoyle, W. Fowler, E. Burbidge and G. Burbidge, *Ap. J.* **139**, 909 (1964).
[21] R. Hagedorn, *Suppl. Nuovo Cimento*, **6**, 311 (1968) and *loc. cit.*
[22] R. Hagedorn, *Journal de Physique*, **30**, Suppl. C3, 79 (1969).
[23] M. Ruderman, *Journal de Physique*, **30**, Suppl. C3, 152 (1969).
[24] D. Kirzhnits, *Soviet Phys. JETP*, **11**, 365 (1960).
[25] E. Salpeter, *Ap. J.* **134**, 669 (1961).
[26] H. Van Horn, *Ap. J.* **151**, 227 (1968).
[27] M. Ruderman, *Nature, London*, **218**, 1128 (1968).
[28] L. Mestel and M. Ruderman, *Mon. Not. R. Astron. Soc.*, **136**, 27 (1967).
[29] H. Van Horn, *Phys. Letters*, **28A**, 706 (1969).
[30] S. Brush, H. Sahlin and E. Teller, *J. Chem. Phys.* **45**, 2102 (1966).
[31] W. Langer, L. Rosen, J. Cohen and A. Cameron, *Astrophys. Spa. Sci.* **5**, 259 (1969).
[32] M. Ruderman, *Nature, London*, **218**, 1128 (1968).
[33] A. Migdal, *Soviet Phys. JETP*, **10**, 176 (1960).
[34] V. Ginzburg and D. Kirzhnits, *Soviet Phys. JETP*, **20**, 1346 (1965).
[35] M. Ruderman, *Proceedings* of the Fifth Eastern Theoretical Physics Conference, D. Feldman, Editor. Benjamin: New York (1967).
[36] R. Kennedy, L. Wilets and E. Henley, *Phys. Rev.* **133**, B 1131 (1964).
[37] M. Hoffberg, A. Glassgold, R. Richardson and M. Ruderman, *Phys. Rev. Letters*, **24**, 775 (1970).
[38] G. Baym, C. Pethick and D. Pines, *Nature, London*, **224**, 674 (1969).
[39] G. Baym, C. Pethick and D. Pines, *Nature, London*, **224**, 673 (1969).
[40] P. Reichley and G. Downs, *Nature, London*, **222**, 229 (1969).
[41] V. Radhakrishnan and R. Manchester, *Nature, London*, **222**, 228 (1969).
[42] P. Boynton, E. Groth, R. Partridge and D. Wilkinson, Princeton Pulsar Symposium (6 November 1969) Proceedings.
[43] G. Baym, C. Pethick, D. Pines and M. Ruderman, *Nature, London*, **224**, 872 (1969).
[44] V. Tkachenko, *Soviet Phys. JETP*, **23**, 1049 (1966).
[45] F. Dyson, Private communication.
[46] M. Ruderman, *Nature, London*, **225**, 619 (1970).
[47] D. Richards, G. Pettengill, G. Counselman and J. Rankin, *I.A.U. Circ. No. 2180* (30 October 1969).